EXERCISES [MAI 2.19]

PERCENTAGE CHANGE – FINANCIAL APPLICATIONS

SOLUTIONS

Compiled by: Christos Nikolaidis

A. Paper 1 questions (SHORT)

PERCENTAGE CHANGE

1. **METHOD A: By using the formula**

(a) $FV = 1000\left(1+\frac{8}{100}\right)^n = 1000(1.08)^n$

(i) $FV = 1000(1.08)^5 = 1469$

(ii) $FV = 1000(1.08)^{-5} = 681$

(iii) Solve for n : $1000(1.08)^n = 2000$, n=9.0064 so n=10

(b) $FV = 1000\left(1-\frac{8}{100}\right)^n = 1000(0.92)^n$

(i) $FV = 1000(0.92)^5 = 659$

(ii) $FV = 1000(0.92)^{-5} = 1517$

(iii) Solve for n: $1000(0.92)^n = 500$, n=8.31 so n=9

METHOD B: By using GDC-financial mode

(a) I% = 8, PV= –1000, [PMT=0, P/Y=1, C/Y=1]

(i) Set n=5 then FV=1469

(ii) Set n= –5 then FV=681

(iii) Set FV=2000 then n= 9.0064 and so n =10

(b) I% = –8, PV= –1000 [PMT=0, P/Y=1, C/Y=1]

(i) Set n=5 then FV=659

(ii) Set n= –5 then FV=1517

(iii) Set FV=500 then n= 8.31 and so n=9

2. (a) 253250 (accept 253000)

(b) 1972 → 2002 is 30 years, increase of 1.3% → 1.013

368000 (accept 368318)

(c) $250000 \times 1.013^n > 400000$

EITHER by **GDC-SolveN** or **Graph**

OR $\Rightarrow 1.013^n > 40/25 \Rightarrow 1.013^n > 1.6$

$\Rightarrow n \log 1.013 > \log 1.6 \Rightarrow n > \log 1.6 / \log 1.013 \Rightarrow n > 36.4$

3. $1.023^t = 2$ **EITHER** **GDC-SolveN**

OR $t = \frac{\ln 2}{\ln 1.023} = 30.48...$

30 minutes (nearest minute) **Note:** Do not accept 31 minutes.

4. (a) $\frac{15.2}{1.027} = 14.8$ million , (OR $15.2(1.027)^{-1}$

(b) $\frac{15.2}{(1.027)^5} = 13.3$ million (OR $15.2(1.027)^{-5}$

FINANCIAL APPLICATIONS

5. METHOD A: By using the formula

(a) (i) $FV = 10000\left(1+\frac{12}{100}\right)^{5} = 17623.42$

(ii) $FV = 10000\left(1+\frac{12}{100\times 2}\right)^{5\times 2} = 17908.48$

(iii) $FV = 10000\left(1+\frac{12}{100\times 4}\right)^{5\times 4} = 18061.11$

(iv) $FV = 10000\left(1+\frac{12}{100\times 12}\right)^{5\times 12} = 18166.97$

(b) (i) $20000 = 10000\left(1+\frac{12}{100}\right)^{n}$ by GDC the solution is $n = 6.11$, so $n = 7$

(ii) $20000 = 10000\left(1+\frac{12}{100\times 12}\right)^{12n}$ by GDC the solution is $n = 5.81$, so $n = 6$

METHOD B: By using GDC-financial mode

(a) (i) n=5, I% = 12, PV = –10000, P/Y=1, C/Y=1. Then press FV = 17623.42
(ii) C/Y=2, then FV = 17908.48
(iii) C/Y=4, then FV = 18061.11
(iv) C/Y=12, then FV = 18166.97

(b) (i) I% = 12, PV= –10000, P/Y=1, C/Y=1, FV=20000. Then press n=6.11 so $n = 7$
(ii) I% = 12, PV= –10000, P/Y=1, C/Y=12, FV=20000. Then press n=5.81 so $n = 6$

6. (a) **METHOD A: By using the formulas**

(i) 10000(1.05) +10000 = 20500

(ii) $10000(1.05)^2 + 10000(1.05) + 10000 = 31525$

(iii) $10000\left(\frac{1.05^{11}-1}{1.05-1}\right) = 142067.87$

[if the last payment is not included we subtract 10000 from each result]

METHOD B: By using GDC-financial mode

I% = 5, PV= –10000, PMT= –10000, P/Y=1, C/Y=1.

(i) Set n=1 then FV gives 20500

(ii) Set n=2 then FV gives 31525

(iii) Set n=10 then FV gives 142067.87

[if the last payment is not included we subtract 10000 from each result]

(b) **By using GDC-financial mode** FV= 200000 gives n = 13.2 so n = 14

(c) The 1st 10000 runs for n years $FV_1 = 10000 \times 1.05^{n}$

The 2nd 10000 runs for n-1 years $FV_2 = 10000 \times 1.05^{n-1}$

…

The last but one 10000 runs for 1 year $FV_n = 10000 \times 1.05^{1}$

The last payment is 10000 $FV_{n+1} = 10000$

The total sum is $10000(1+1.05+1.05^2+1.05^3+ \dots + 1.05^n) = 10000 \times \left(\frac{1.05^{n+1}-1}{1.05-1}\right)$

[if the last payment is not included we subtract 10000 the result]

7. (a) **METHOD A: By using the formulas**

(i) 10000(1.05)+1000 =11500

(ii) 10000×1.05^2 +[$1000\times1.05+1000$] =13075

(iii) $10000\times1.05^{10}+1000\times\left(\dfrac{1.05^{10}-1}{1.05-1}\right)=28866.84$

[if the last payment is not included we subtract 1000 from each result]

METHOD B: By using GDC: financial

I% = 5, PV= –10000, PMT= –1000, P/Y=1, C/Y=1.

(i) Set n=1 then FV gives 11500

(ii) Set n=2 then FV gives 13075

(iii) Set n=10 then FV gives 28866.84

[if the last payment is not included we subtract 1000 from each result]

(b) **By using GDC: financial** FV= 50000 gives n = 17.36 so n = 18

(c) The 1st 10000 runs for n years 10000×1.05^n

The 1st 1000 runs for n-1 years $1000\times1.05^{n-1}$

…

The last but one 1000 runs for 1 year 1000×1.05^1

The last payment is 1000 1000

Sum of the last n terms $=1000(1+1.05+1.05^2+1.05^3+\ldots+1.05^{n-1})=1000\times\left(\dfrac{1.05^n-1}{1.05-1}\right)$

$$FV=10000\times1.05^n+1000\times\left(\frac{1.05^n-1}{0.05}\right)$$

[if the last payment is not included we subtract 10000 the result]

8. **By using GDC: financial**

(a) I% = 5, PV= –10000, PMT= –1000, P/Y=1, C/Y=12.

(i) Set n=1 then FV gives 11511.62

(ii) Set n=10 then FV gives 29116.41

[if the last payment is not included we subtract 1000 from each result]

(b) **By using GDC: financial** FV= 50000+1000 gives n = 17.15 so n = 18

9. **By using GDC: financial**

(a) I% = 5, PV= –10000, PMT= –100, P/Y=12, C/Y=12.

(i) Set n=12 then FV gives 11739.50

(ii) Set n=120 then FV gives 31998.32

[if the last payment is not included we subtract 100 from each result]

(b) **By using GDC: financial** FV= 50000 gives n = 187.04 so n = 188 months

10. **By using GDC: financial**

I% = 5, PV= –10000, P/Y=1, C/Y=1

(a) $10000\times\dfrac{5}{100}=500$ euros

(b) PMT = +500 , FV = 10000 (since we simply receive the interest of $500)

(c) PMT = +400, FV = 11257.79

(d) PMT = +1000, FV = 3711.05

(e) FV=0, then n=14.2 so after 15 years, For n=15, FV = -789.28 so L = 210.72

11. $n = 10$, I% = 15, PV= 20000, FV=0, P/Y=1, C/Y=1

(a) PMT = –3985.04 so the repayment is 3985.04 per year

(b) $3985.04 \times 12 = 39850.04$

(c) $39850 - 20000 = 19850.40$

12. $n = 10 \times 12 = 120$, I% = 15, PV= 20000, FV=0, P/Y=12, C/Y=12

(a) PMT = –322.67 so the repayment is 322.67 per month

(b) $322.67 \times 120 = 38720.40$

(c) $38720.40 - 20000 = 18720.40$

13. $n = 10 \times 4 = 40$, I% = 15, PV= 20000, FV=0, P/Y=4, C/Y=4

(a) PMT = –973.19 so the repayment is 973.19 per quarter

(b) $973.19 \times 40 = 38927.60$

(c) $38927.60 - 20000 = 18927.60$

14. $n = 10 \times 12 = 120$, PMT= , PV= 20000, FV=0, P/Y=12, C/Y=12

(a) I% = 13.12 so the interest rate is 13.12% (per year compounded monthy)

(b) $300 \times 120 = 36000$

(c) $36000 - 20000 = 16000$

15. $5000(1.063)^n > 10000$ (or $(1.063)^n > 2$)

METHOD A: trial and error by a GDC

A good way of communicating this is suggested below.

When $n = 11$, $(1.063)^n = 1.9582$, when $n = 12$, $(1.063)^n = 2.0816$ Hence $n = 12$ years

METHOD B: using logarithms

$n \log(1.063) > \log 2 \Rightarrow n > 11.345...$ 12 years

16. (a) $\$\ 1000 \times 1.075^{10} = \$\ 2061$ (nearest dollar)

(b) $1000(1.075^{10} + 1.075^{9} + ... + 1.075) = \dfrac{1000(1.075)(1.075^{10} - 1)}{1.075 - 1} = \$\ 15\ 208$ (nearest dollar)

17. 15% per annum = $\dfrac{15}{12}\% = 1.25\%$ per month

Total value of investment after n months, $1000(1.0125)^n > 3000 \Rightarrow (1.0125)^n > 3$

$n \log(1.0125) > \log(3) \Rightarrow n > \dfrac{\log(3)}{\log(1.0125)}$

Whole number of months required so $n = 89$ months. [or directly by GDC]

18. (a) Value = $1500(1.0525)^3 = 1748.87 = 1749$ (nearest franc)

(b) $3000 = 1500(1.0525)^t \Rightarrow 2 = 1.0525^t$

$t = \dfrac{\log 2}{\log 1.0525} = 13.546$ It takes 14 years.

(c) $3000 = 1500(1 + r)^{10}$ or $2(1 + r)^{10}$

$\Rightarrow \sqrt[10]{2} = 1 + r$ or $\log 2 = 10 \log(1 + r)$

$\Rightarrow r = \sqrt[10]{2} - 1$ or $r = 10^{\frac{\log 2}{10}} - 1$

$r = 0.0718$ [or 7.18%]

B. Paper 2 questions (LONG)

19. (a) (i) N = 24

I % = 14

PV = −14000

FV = 0

P/Y = 4 **Then (€)871.82**

(ii) 4×6×871.82 = (€)20923.68

(iii) 20923.68 −14000 = (€)6923.68

(b) (i) 0.9×14000 (=14000 − 0.10×14000) = (€)12600.00

(ii) N = 72

PV =12600

PMT = −250

FV = 0

P/Y = 12

C/Y = 12 **Then 12.56(%)**

(c) **EITHER** Bryan should choose Option A, no deposit is required

OR Bryan should choose Option B

cost of Option A (6923.69) > cost of Option B (72 × 250 − 12600 = 5400)

(d) real interest rate is 0.4 − 0.1 = 0.3%

value of other payments 250 + 250×1.003+…+ 250×1.00371 = 20 058.43

OR financial app on a GDC

value of deposit at the end of 6 years 1400× 1.003 =1736.98

Total value is (€) 21 795.41

N = 72 (6×12)

I % = 3.6 (0.3×12)

PV = 0

PMT = −250

FV =

P/Y = 12

C/Y = 12

OR

N = 72 (6×12)

I % = 0.3

PV = 0

PMT = −250

FV =

P/Y = 1

C/Y = 1

20. (a) (i) n = 1, I% = –1.1, PV= –7000, P/Y=1, C/Y=1

Then FV = 6923

(ii) n = 12, I% = –0.1, PV= –7000, P/Y=1, C/Y=1

Then FV = 6916.46

OR

n = 1, I% = –1.2, PV= –7000, P/Y=1, C/Y=12

Then FV = 6916.46

(b) (i) n = 5, I% = –1.1, PV= –7000, P/Y=1, C/Y=1

Then FV = 6623.38

(ii) n = 60, I% = -0.1, PV= –7000, P/Y=1, C/Y=1

Then FV = 6592.15

OR

n = 5, I% = –1.2, PV= –7000, P/Y=1, C/Y=12

Then FV = 6592.15

(c) n = 5, I% = 12, PV= –7000, P/Y=1, C/Y=12

Then FV = 12716.88

(e) Per month Interest rate 1% inflation rate 0.1%

hence real interest rate = 1-0.1=0.9%

n = 5×12=60, I% = 0.9, PV= –7000, P/Y=1, C/Y=1

Then FV = 11983.07

OR

Per year interest rate 12% inflation 0.1×12 = 1.2%

hence real interest rate = 12-1.2=10.8% (compounded mothly)

n = 5, I% = 10.8, PV= –7000, P/Y=1, C/Y=12

Then FV = 11983.07

(e) I% = 12, PV= –7000, PMT=150, P/Y=12, C/Y=12

(a) n = 5×12=60. Then FV = 466.43

(b) If FV = 0 then n=63.17 so at the end of the 64^{th} month. That is end of April 2025

OR Trial and error

For n=63 FV=26.04

For n=64 FV= –123.69

So at the end of April he will receive the last 150-123.69 =26.31 euros